Photonic Crystal Negative Refractive Optics

Toshihiko Baba¹,²,*, Hiroshi Abe¹, Tomohiko Asatsuma¹,², and Takashi Matsumoto¹,²

¹ Department of Electrical and Computer Engineering, Yokohama National University, 79-5 Tokiwadai, Hodogayaku, Yokohama 240-8501, Japan
² Core Research for Evolutional Science and Technology (CREST), Japan Science and Technology Agency (JST), 5, Sanbancho, Chiyodaku, Tokyo 102-0075, Japan

Photonic crystals (PCs) are multi-dimensional periodic gratings, in which the light propagation is dominated by Bragg diffraction that appears to be refraction at the flat surfaces of the PC. The refraction angle from positive to negative, perfectly or only partially obeying Snell's law, can be tailored using photonic band theory. The negative refraction enables novel prism, collimation, and lens effects. Because PCs usually consist of two transparent media, these effects occur at absorption-free frequencies, affording significant design flexibility for free-space optics. The PC slab, a high-index membrane with a two-dimensional airhole array, must be carefully designed to avoid reflection and diffraction losses. Light focusing based on negative refraction forms a parallel image of a light source, facilitating optical couplers and condenser lenses for wavelength demultiplexing. A compact wavelength demultiplexer can be designed by combining the prism and lens effects. The collimation effect is obtainable not only inside but also outside of the PC by optimizing negative refractive condition.

Keywords: Photonic Crystal, Super-Prism, Super-Lens, Super-Collimation, Negative Index, Negative Refraction, Dispersion Surface.

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1. INTRODUCTION

Metamaterials consisting of metal/dielectric composites have attracted much attention recently because they can produce negative refraction of light and related optical phenomena such as super-focusing beyond the diffraction limit and cloaking. However, their narrow operating bandwidth (due to the required electromagnetic resonance) limits applications, and large absorption loss in the metal is a severe concern for optical propagation. Negative refraction also occurs in photonic crystals (PCs), due to a principle completely different from that of metamaterials. PCs are artificial multi-dimensional structures having a period on the order of an optical wavelength and provide various optical functionalities. They exhibit a photonic bandgap, i.e., a forbidden frequency band for optical modes, which can be used for omnidirectional reflectors in cavities and waveguides. Considering them instead to be a kind of diffraction grating, they should exhibit unique characteristics due to their multi-dimensionality, such as negative refraction. Because PCs consist of pure dielectric composites, light propagation occurs without absorption losses. In addition, they operate in a wide bandwidth range unless the diffraction is intentionally designed to be wavelength selective.

The diffraction characteristics of PCs have been investigated since the 1980s, for example, by Russell et al.¹ Kosaka et al. in 1998² explained the diffraction in terms of the dispersion surfaces of photonic bands (which they called “Bose surfaces” in analogy to the Fermi surfaces of metals). They predicted prism, lens, and collimation effects based on the diffraction which appears to be the negative refraction for limited angles of the incident light.³ In 2000, Notomi theoretically showed that negative refraction can occur for all incident angles in a limited part of dispersion surfaces and that a negative refractive index can be defined.⁴ Based on these studies, other groups have
discussed these phenomena and their device applications for spectrometers, dispersion compensators, beam scanners, polarizers, sensors, and so on.\textsuperscript{5–39} In many studies, however, compelling experimental demonstrations were limited by the following three issues:

1. fabrication difficulties of PCs suitable for negative refraction,
2. suppression of unwanted reflection and diffraction at the PC surfaces, and
3. observation of light propagation inside and outside the PCs. The first issue was solved by the use of a PC slab, a high-index membrane with an airhole array. It is fabricated on a commercially available silicon-on-insulator (SOI) wafer or III–V compound semiconductor wafer by etching airholes into a high-index top layer of typically 200-nm thickness, and selectively removing the second layer. Light is confined in the slab by total internal reflection and is affected by the PC effect in the plane.

Dr. Toshihiko Baba received his Ph.D. Degree from the Division of Electrical and Computer Engineering, Yokohama National University, in 1990. He became an associate professor and a full professor at the same university in 1994 and 2005, respectively. Over the past 25 years, Dr. Baba has conducted research on numerous micro/nano-photonic devices, including ARROW waveguides, VCSELs, microdisk lasers, photonic crystals, and Si photonics. In particular, he reported many pioneering findings on photonic crystal waveguides, slow light devices, nanolasers, negative refractive optics, and Si photonic wire devices. Dr. Baba is a member of IEICE, JSAP, IEEE/Photonics, APS, and OSA. He has received nine academic awards from societies and conferences, and served as a distinguished lecturer of the IEEE/LEOS (2006–7).

Hiroshi Abe

Tomohiko Asatsuma

Takashi Matsumoto
In this paper, we present the theoretical design and experimental demonstrations of negative refractive effects in a PC slab, and show how the other issues have been resolved. We first focus on the prism and lens effects and describe a compact wavelength demultiplexer as an application of them. We also discuss the collimation effect, which is usually expected inside PCs but is also obtainable outside of the PC by employing a slight negative refractive condition.

2. DISPERSION SURFACE AND NEGATIVE REFRACTION

A photonic band diagram plots eigenfrequency \( \omega \) (or its normalized form \( a\omega/2\pi c = a/\lambda \), where \( a \) is the lattice constant of the PC) as a function of the wavevector \( \mathbf{k} \) for select paths in the Brillouin zone. The dispersion surface displays equi-frequency contours of each band in the Brillouin zone.\(^2\) Figure 1 shows schematic behaviors of light and of the corresponding dispersion surfaces in a two-dimensional (2D) PC consisting of a triangular lattice. Unique shape contours of the higher order bands arise from multi-dimensional zone folding into the first Brillouin zone. According to optical theory, the group velocity of light \( v_g \) is \( d\omega/dk \), meaning that \( v_g \) is proportional to the band slope. Therefore, the gradient of the dispersion surface indicates the orientation of the Poynting vector \( \mathbf{S} \) in the PC. A particular wave on the dispersion surface can be excited from outside the PC so that energy and momentum conservation laws are satisfied at the PC surface. The frequency contour of the dispersion surface and the incident \( \mathbf{k} \) vector are uniquely determined by the wavelength \( \lambda \) and angle \( \theta_{in} \) of the incident light and by the refractive index of the medium outside the PC. The momentum is conserved parallel to the PC surface. The excited wave is indicated by the intersection of the contour and the \( \mathbf{k}/\lambda \) line shown in Figure 1. The \( \mathbf{S} \) vector exhibits negative refraction for the dispersion surface of a higher order band, which is folded into the first Brillouin zone and has a slope toward the zone center (\( \Gamma \) point). When the contour shows a sharp slope or a flat shape, \( \mathbf{S} \) becomes sensitive or insensitive to \( \lambda \) and \( \theta_{in} \), respectively. These are called prism and collimation effects. (The collimation effect occurs for the critical condition between positive and negative refraction.) For an inwardly curving contour, \( \mathbf{S} \) shows directions of focusing called the lens effect. One could define a negative refractive index for these effects from Snell’s law. Such a negative index is mainly useful when the negative refraction uniformly occurs for all \( \theta_{in} \). Such a condition appears on the circular contours in a limited frequency range near the \( \Gamma \) point.\(^4\) This condition is always inside the light cone in the PC slab, for which light is not confined in the slab but is instead radiated into out-of-plane directions.

Figure 2(a) shows details of the dispersion surface of the second band in an actual PC slab designed for prism and lens effects.\(^{18} \) Here, a square lattice rotated by 45° is employed because unwanted reflection and diffraction losses at surfaces are reduced for interface structures, as shown in Section 3. When \( \theta_{in} \) is fixed at 10°, as shown by the appropriate equi-incident-angle line, the PC prism exhibits frequency dependent negative refraction. When diverging light centered at \( \theta_{in} = 0° \) with a width \( |\Delta\theta_{in}| \leq 7° \) is incident on the PC lens, it is focused by negative refraction. All of these conditions lie outside the light cone. Figure 2(b) shows those for the collimation effect. In this case, a square lattice with no rotation is employed as it allows low loss transmission of light for \( |\Delta\theta_{in}| \leq 7° \) even though PC surfaces do not have particular interface structures. Light collimation is expected for \( |\Delta\theta_{in}| \leq 7° \) and \( a/\lambda = 0.32–0.34 \), for which \( \mathbf{S} \) vectors lie on the flat part of equi-frequency contours. Note that, due to small distortions in the contours, perfect collimation cannot be achieved but partial collimation in an effective length.

Fig. 1. Schematics of light propagation in a 2D PC (upper figures) and corresponding dispersion surfaces (green lines in the lower figures). The black arrows denote incident \( \mathbf{k} \) vectors and the orange arrows denote \( \mathbf{S} \) vectors in the PC. (a) Positive refraction, (b) prism effect by negative refraction, (c) collimation effect, and (d) lens effect by negative refraction.
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3. INTERFACE STRUCTURES

When simulating the lens and prism effects at a PC surface using the finite-difference time-domain (FDTD) method, light does not easily transmit through the PC due to the reflection and diffraction. They are not serious when a low-index medium such as air is outside the PC and the PC itself has a low effective index such as if it consists of narrow pillars. Many theoretical studies have therefore assumed such conditions. In practice, however, it is difficult to experimentally demonstrate light propagation and negative refraction in such a structure because the light is easily radiated into out-of-plane directions. The 2D optical confinement is achieved by a combination of a high-index slab and a PC, but is subject to strong reflection and diffraction losses that perturb the negative refraction, as shown for the FDTD simulation in Figure 3(a). Light enters from a high-index medium to the PC having airholes. Therefore, total internal reflection occurs locally at the airhole boundaries and enhances the reflected waves. The total loss at the input and output (I/O) interfaces is typically 10–20 dB. In addition, the modal mismatch on the PC surface between an incident wave and a target Bloch wave in the PC results in the excitation of Bloch waves satisfying momentum conservation in the higher order Brillouin zones. To excite the target wave efficiently, some optimum interfacial structure that compensates this mismatch is necessary.

Two types of interfaces have been studied, adiabatic tuning and local tuning of the structural parameters. For the former, the airhole diameters gradually increase from the high-index slab to the PC slab. This method is not very effective because the frequency contour changes discontinuously at the photonic bandgap and at...
the local bandgaps on an equi-incident-angle line.\textsuperscript{10} Local bandgaps could be avoided and the continuity ensured by a simultaneous change of the airhole diameter and shape and the lattice constant and topology. However, the full photonic bandgap between the first and second bands cannot be avoided. Besides, the adiabatic structure can become longer than the PC itself, which is undesirable for compact photonic devices and circuits. Regarding the latter tuning method, some modifications of the airhole shape on the PC surface are effective. Figures 3(b and c) show FDTD simulations of prism and lens effects. Here, the interfacial airhole shape is optimized so that their losses are minimized for a nearly collimated Gaussian beam with a center angle $\theta_{\text{in}} = 10^\circ$ in the case of the prism effect and a diverging Gaussian beam with $\theta_{\text{in}} = 0^\circ$ and a maximum $|\Delta \theta_{\text{in}}| = 7^\circ$, respectively. Their losses are then suppressed to 0.20 and 0.56 dB/interface, respectively.\textsuperscript{16}

4. PRISM EFFECT

In general, negative refraction occurs a higher bands having a slope toward the $\Gamma$ point. However, it is difficult to design a low-loss interface when the band order is high and the target Bloch waves have complicated profiles with high spatial frequency components. Therefore, the second band is often employed. To observe the prism effect, a point on the dispersion surface must be carefully selected to balance a large angular dispersion, indicated by the wavelength sensitivity parameter $q = d\theta_{\text{i}}/d(a/\lambda)$, and a good beam quality, indicated by the beam collimation parameter $p = d\theta_{\text{i}}/d\theta_{\text{in}}$ where $\theta_{\text{i}}$ is the refraction angle in the PC.\textsuperscript{10} Figure 4 displays these parameters in the Brillouin zone for the dispersion surface of Figure 2(a). The angular dispersion becomes extremely large in the lateral direction up to the $\Gamma$ point, and the beam quality is degraded. Because the light beam is widely diverging in the PC, negative refraction is difficult to confirm. The dispersion is relatively large and a constant beam quality is only maintained in the dotted circle for $\theta_{\text{in}} \sim 10^\circ$.

![Fig. 4](image)

**Fig. 4.** Characteristic parameters of a PC slab prism. The vertical lines denote equi-incident angles $\theta_{\text{in}}$ of every 5°. The central black circles are inside the light cone.

The device was fabricated on a SOI substrate, as shown in Figure 5(a). In the measurement, light from a tunable laser was coupled to a 0.5-μm-wide single-mode Si photonic wire waveguide, which was integrated on the same substrate. The beam width of the guided light was appropriately expanded at the end of the waveguide by a taper to limit the divergence angle $\Delta \theta_{\text{in}}$ of the incident beam. The wavelength $\lambda$ of the light was initially tuned to 1.37 μm, satisfying the light-cone condition. Negative refraction was then observed by looking down onto the PC using an InGaAs camera, as shown in Figure 5(b).\textsuperscript{18} Although such images could not be observed for $\lambda > 1.39$ μm, which is outside the light cone, negative refraction was confirmed for such long wavelengths from the light scattering at the end of the PC. Figure 2(c) shows the angular dispersion for three samples with different airhole diameters $2r$. The refraction angle varied in the range $46^\circ$–$61^\circ$ over a wavelength variation of 60 nm. The experimental plots agree well with theoretical curves obtained using the FDTD method.

5. LENS EFFECT

Similarly to the prism effect, the lens effect was observed in a PC slab based on the dispersion surface of
Figure 2(a). Light is output from the Si photonic wire waveguide with the condition $\theta_{in} = 0^\circ$ and $|\Delta \theta_{in}| < 7^\circ$, expanded in the Si slab waveguide, and incident on the PC with $a = 0.45 \mu m$ and $2\tau = 0.28 \mu m$, which has an optimized interface shown in Figure 6(a). Figure 6(b) shows a ray tracing obtained from the gradient of a contour curve at $a/\lambda = 0.30$. The ray tracing reveals that the focal spot is degraded by spherical aberrations and the focal length increases with $\theta_{in}$. The spot width, defined as the minimum width between rays, is $4.0a = 1.2\lambda$. Figure 6(c) shows a streak image of light propagating in the fabricated device at $\lambda = 1.305 \mu m$ ($a/\lambda = 0.34$) within the light cone. Light focusing is observed with a strong intensity at the focal point. Unlike the ray tracing, in which a point light source was assumed, the focal spot in the experiment is limited by the modal width of the source. In the present case, it was $1.1 \mu m (= 2.3a = 0.8a)$ at the tapered end of the Si photonic waveguide. But the observed spot width was $2.0 \mu m (= 4.4a = 1.5\lambda)$, which is close to that in the ray tracing. This suggests that the spot is dominated by aberrations.

A parabolic equi-frequency contour would eliminate the aberration. Some PC structures exhibit such a contour for paraxial rays of $|\Delta \theta_{in}| < 3^\circ$. To expand the range of $\Delta \theta_{in}$, we propose a composite PC, in which the standard PC is cascaded to a second PC (designated PC) having opposite aberration characteristics so that the net aberration is zero. A shortening of the focal length at wider $\Delta \theta_{in}$ is needed for PC. Numerical explorations led to a structure consisting of triangular lattice of elliptical airholes, as shown in the lower half of Figure 6(d). For the smooth propagation of Bloch waves across the boundary between the two PCs, the airhole diameter and shape were adiabatically changed while maintaining the same airhole pitch in the lateral direction. Figure 6(e) shows a ray tracing in the composite PC for $|\Delta \theta_{in}| \leq 7^\circ$. Here, the length of the first PC was optimized to minimize the aberration. The spot width was $0.86a = 0.26a$, which is 4.7 times smaller than that for the single PC and is smaller than the diffraction limit, demonstrating suppression of the aberration in the composite PC. A streak image for the fabricated device at $\lambda = 1.325 \mu m$ ($a/\lambda = 0.35$) is shown in Figure 6(f). The focal spot narrowed, as compared with that for the single PC. Light rapidly decayed beyond the spot, possibly due to stronger radiation of the light cone in PC which had larger airholes. The spot width was $1.4 \mu m$ ($= 3.0a = 1.1\lambda$). Although still larger than the mode size of the light source, the aberration is estimated to be three times smaller than that for the single PC.

When a PC has parallel I/O ends, light diverging beyond the focal point is refocused outside of the PC. In other words, the image of the light source is transferred to the far side of the PC. As a demonstration, we characterized the coupling of light from an input waveguide to output waveguides, as shown in Figure 7(a). Here, the length of the PC was twice as long as the focal length, i.e., $10.6 \mu m$ for $\lambda \approx 1.5 \mu m$, so that the focal point was located at the center of the PC. The output interface was symmetric with respect to the input interface, with the output waveguides arranged in parallel. Focusing inside the PC was observed at $\lambda \leq 1.34 \mu m$. Next, the light refocused outside the PC and was extracted from the center output waveguide. The same refocusing occurred at $\lambda > 1.34 \mu m$. When using a positive refractive lens, images of multiple objects are formed at anti-symmetric positions with respect to the lens. The distance from the objects to the images is longer than twice the focal length of the lens. To form an image of a large object, a large lens is needed.

![Figure 6](image6.png)

**Fig. 6.** Experimental observation of the lens effect by negative refraction. (a) Fabricated PC lens with deformed airholes at the PC surface for low loss transmission. (b) and (c) A ray tracing and a streak image with intensity profiles of the propagating light for the PC lens, respectively. (d) Fabricated composite lens consisting of a cascaded PC and a PC having opposite aberration characteristics. (e) and (f) A ray tracing and a streak image of the propagating light for the composite lens, respectively.

![Figure 7](image7.png)

**Fig. 7.** Demonstration of novel focusing by a PC lens. (a) Refocusing outside of the PC at two different wavelengths inside and outside the light cone. The arrows indicate the position of the output light. (b) Parallel focusing of four light sources.
with a long imaging distance. However, in the case of a negative refractive lens, images are formed at symmetric positions. The distance between the objects and images is shortened by placing the objects close to the lens. Any size of objects are imaged if the lens has a size similar to that of the objects. To demonstrate these properties, we observed parallel focusing of multiple light sources: the input waveguide was divided into four branches. Figure 7(b) shows a streak image at $\lambda = 1.30 \mu m$. The image of each light source was formed at a symmetrical position inside the PC. The focal spot widths and focal lengths are approximately uniform and constant over the wavelength range from 1.265–1.300 $\mu m$. Ray tracing and FDTD simulations agree with these experimental results.

6. APPLICATION TO DEMULTIPLEXERS

The PC prism and lens achieve a large angular dispersion and parallel focusing, respectively. By using the former as a diffraction grating and the latter as a condenser lens, a compact wavelength demultiplexer can be formed, as depicted in Figure 8(a). The fundamental design of the demultiplexer is different from that of conventional demultiplexers, because the components utilize the negative refraction of the $\mathbf{S}$ vector, not of the $\mathbf{k}$ vector. Here, light from the Si wire waveguide is tilted by $10^\circ$ relative to the normal and is incident on a PC prism having parallel surfaces. The output angle from the prism is the same as the incident angle, since the tangential component of the incident $\mathbf{k}$ vector is conserved. Light beams having different wavelengths are spatially separated in the prism and are incident at different positions on the PC lens. The lens is tilted by $10^\circ$ relative to the super prism, so that the output light from the prism is oriented normal to the flat surface of the lens. The structural parameters of the PC lens are slightly modified from those described above to adjust the operating wavelength to $\lambda \approx 1.55 \mu m$, while maintaining a dispersion surface similar to that of Figure 2. The output surface of the PC lens is curved to compensate for chromatic aberration; the length of the lens at each lateral position is equal to the focal length of the corresponding wavelength. The lens is connected to output channel waveguides that are terminated by rectangular airholes that collect the output light. Figure 8(b) shows the near-field patterns of laser light at different wavelengths output from different ports. Transmission spectra were measured for another sample, as shown in Figure 8(c).

To avoid the resonance inside the output waveguide, we used incoherent amplified spontaneous emission source and optical spectrum analyzer for the measurement. The channel spacing of the four output waveguides is 11 nm, which is narrow for such a small device ($80 \times 100 \mu m^2$, excluding the input and output waveguides). It can be further narrowed by enlarging the prism; a spacing of 0.4 nm is theoretically predicted for a device size of 1 mm². On the shorter wavelength side, the spectral width and channel spacing increase to 15–30 nm due to the lower angular dispersion. The channel spacing can be equalized by elongating the beam separation length of the super prism on the shorter wavelength side.

7. COLLIMATION EFFECTS

In some previous studies, the collimation effect has been observed in PCs which show the flat equi-frequency
contours like those shown in Figure 2(b). We observed this in the PC slab through the light cone at \(\lambda = 1.415 \mu m\), as shown in Figure 9, where \(a = 0.55 \mu m\) and \(2r = 0.28 \mu m\). Even though the PC surface is simply terminated by circular airholes, light incidence with a 0.5 dB loss was estimated for \(|\Delta \theta_m| \leq 7^\circ\) in the FDTD simulation. In this experiment, on the other hand, wider angular components should be excited at the end of the input waveguide. However, light scattering at the PC surface was not strong. The streak image suggests straight light propagation of the collimation effect, although it disappears inside the PC slab due to the radiation loss of the light cone. This loss should be eliminated at wavelengths outside the light cone. Such a collimation effect is useful for crossing multiple light beams without interactions.

In this section, we further discuss the collimation effect outside of the PC. Since \(k_z\) is always conserved, pure collimation is not expected outside the PC. A diverging light beam similar to the one incident on the PC should be excited at the output surface when the same medium surrounds the PC. However, this expectation is not always the correct. Effective collimation occurs when a light source is located close to the PC surface and the flat part of equi-frequency contours exhibits slight negative refraction. When assuming \(a/\lambda = 0.33–0.35\), incident light components of \(|\Delta \theta_m| \leq 11–7^\circ\) can be coupled with the flat part, respectively. Figure 10(a) shows the ray tracing of light with \(|\Delta \theta_m| \leq 7^\circ\) for \(2r/a = 0.57\) and \(a/\lambda = 0.34\). Here, a point source is located only 2.2a apart from the PC surface. Because of this short distance and the slight negative refraction condition at the normalized frequency, incident light is once focused immediately and slowly diverges in the PC. The slight negative refraction occurs again for the so-diverged and expanded beam at the output surface, and so the light is focused outside the PC. But this second focal point is distant from the output surface, affected by the asymmetric location of the first focal point in the PC. Consequently, the effective collimation is obtained within a distance roughly approximated as the second focal length. It is confirmed in the FDTD simulation, as shown in Figure 10(b). Here, A shows the light propagation from a point source toward the free space without PC, while B shows the same situation as that in Figure 10(a). In contrast to isotropic radiation in A, light is collimated by the PC in B, which is maintained even outside the PC. Half light from the point source reaches the PC surface. The coupling efficiency of such light into the collimated light was calculated to be 8.3%. This value increases up to 79% by narrowing the excited light beam so that \(|\Delta \theta_m| \leq 7^\circ\), as shown in C. As understood from the above-mentioned principle of the effective collimation, the light behaviors are sensitive to the position of the light source. When the source is located apart from the PC surface, the first focal point shifts to inside the PC. If the focal point reaches the center of the PC, light appears to be focused rather than collimated outside the PC. If the first focal length is longer than the PC length, neither the first focusing nor the second focusing occur and the light simply diverges. The effective collimation length also depends on the length of the PC. A long PC increases the asymmetry of the location of the first focal point, resulting in a long second focal length and a long effective collimation length.

8. SUMMARY

Negative refraction in a PC slab can be designed and observed when the interface structure of the PC is appropriately optimized. The prism effect shows a large angular
dispersion, usable as a diffraction grating. The lens effect allows real image formation and image transfer, which enable a different imaging system configuration. The collimation effect suppresses the diffraction of light within a finite length even outside of the PC. Regarding the lens effect, super-focusing is a remaining issue. A small spot beyond the diffraction limit, or it is a small source is completely transferred through the PC to form a small spot beyond the diffraction limit, or it is a small source is completely transferred through the angular dependence of transmission and experimentally near the output surface of the PC. However, it seems unclear if the localized field of the point source is completely transferred through the PC to form a small spot beyond the diffraction limit, or it is a small spot apertured by the angular dependence of transmission loss through the PC. Cloaking in a PC, similar to that in metamaterials but free from absorption loss, is of great interest. Such novel light propagation could be realized by the synthesis of certain PCs and dispersion surfaces.

References and Notes


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